Comprehensive Exam on Data Structures and Algorithms: Fall 2008

Instructions: Choose any four of the following six questions to answer. Clearly mark which question you want graded on the front of the exam by placing an X in the appropriate slot.

Problem	1	2	3	4	5	6
Graded?						

Note that you will be graded not just on the answers, but also on work. An answer alone will not be worth much if anything at all. No books, calculators, notes, talking, etc. Using any communications device at all (cell phones, walkie talkies, etc.) will be an automatic failing grade.

WARNING: If you do not mark the questions you want graded or mark them ambiguously (i.e. if you decide to mark 5 questions rather than 4, etc.), we will grade your lowest 4 questions.

1. (a) Construct a Huffman code for the following data: $A \to 0.4$, $B \to 0.1$, $C \to 0.2$, $D \to 0.15$, $E \to 0.15$.

   (b) Encode $ABACABADEA$ using your code.

   (c) Decode 100010111001010.

2. Estimate how many times faster an average successful search will be in a sorted array of $100,000$ elements if it is done by binary search versus sequential search.

3. Using pseudocode, describe an $O(n)$ recursive algorithm that takes as input the root of a binary search tree having n integer keys, and has the effect of printing the keys in sorted order (least to greatest).

4. A k-coloring of an undirected graph $G = (V, E)$ is a function c that maps V to $\{1, 2, \ldots, k\}$, such that, for any edge $(u, v) \in E$, $c(u) \neq c(v)$. Provide an efficient (polynomial-time) algorithm for determining a 2-coloring for a graph if one exists, and prove that it is efficient. Hint: you may assume that G is connected.

5. SET-PARTITION is the problem of determining for a given set S of integers if there exists a subset A of S for which

$$\sum_{x \in A} x = (\sum_{x \in S} x)/2.$$

Similarly SUBSET-SUM is the problem of determining for a given set S of integers and an integer m if there exists a subset A of S for which

$$\sum_{x \in A} x = m.$$

Assuming that SUBSET-SUM is NP-complete, prove that SET-PARTITION is NP-complete.

6. Show how to multiply the complex numbers $a + bi$ and $c + di$ using only three real multiplications. The algorithm should take $a, b, c,$ and d as input, and produce the real component $ac - bd$ and the imaginary component $ad + bc$.