

Comprehensive Exam in Advanced Algorithms :
Spring 2003

1. Prove **briefly** that the following statements are true.

(a) $f(n) = (\ln n)^{\ln n}$ is superpolynomial but not exponential.

(b) $g(n) = (3 - e^{-n})^n$ is exponential.

(c) $h(n) = \sum_{i=1}^n (\frac{3}{i} + \frac{1}{3^i})$ is $O(\ln n)$.

2. Assume that you are given a number of jobs each of which lasts exactly one hour in length. For each job that you manage to accomplish, you will receive money. However, you cannot work on more than one job at any given time. Associated with each job is a deadline for completion. If you do not complete the job by the deadline, you cannot collect the profits associated with the job. (You do not have to complete the job *at* the deadline, just sometime before the deadline.) You only have from time 0 to 7 to work on jobs. Below is a table of job names, deadlines, and profits upon completion. Provide an optimal (maximal profit) schedule for the following list.

Job Name	Deadline	Profit
A	4	\$40
B	3	\$50
C	1	\$20
D	4	\$30
E	3	\$50
F	1	\$30
G	4	\$40
H	7	\$10
I	7	\$60
J	2	\$20

3. Find the optimal parenthesization for the following matrix multiplication.

$$M_1 M_2 M_3 M_4 M_5$$

The sizes of the matrices are as follows: $M_1 : 5 \times 10$, $M_2 : 10 \times 3$, $M_3 : 3 \times 12$, $M_4 : 12 \times 5$, $M_5 : 5 \times 50$

4. The longest simple cycle problem (LSCP) is the problem of finding the longest simple cycle (a cycle that has no repeated vertices other than the beginning and end of the cycle) in a given undirected connected graph. Prove that LSCP is NP-complete. (HINT: Think Hamiltonian.)