Comprehensive Exam on Data Structures and Algorithms: Spring 2007

Instructions: Choose any four of the following six questions to answer. Clearly mark which question you want graded on the front of the exam by placing an X in the appropriate slot.

Problem	1	2	3	4	5	6
Graded?						

Note that you will be graded not just on the answers, but also on work. An answer alone will not be worth much if anything at all. No books, calculators, notes, talking, etc. Using any communications device at all (cell phones, walkie talkies, etc.) will be an automatic failing grade.

WARNING: If you do not mark the questions you want graded or mark them ambiguously (i.e. if you decide to mark 5 questions rather than 4, etc.), we will grade your lowest 4 questions.

1. (a) State Kruskal's greedy algorithm for finding the Minimum Spanning Tree of a weighted undirected graph.

   (b) Assume that you are given a weighted undirected graph $G = (V, E)$. Assume that the vertices of the graph are partitioned into two sets, V_1 and V_2. Furthermore, you are given two sets of edges, E_1 and E_2, such that E_1 is a set of edges of minimum total weight than spans V_1 and E_2 is a set of edges of minimum weight than spans V_2. Show that, if G is connected, then a minimum spanning tree of the graph G can be constructed by using all edges in E_1, E_2, and a single additional edge e^* such that e^* is the edge of smallest weight that has a vertex in V_1 and a vertex in V_2.

   (c) Prove that Kruskal's greedy algorithm always produces a minimum spanning tree correctly.

2. Describe a binary search tree with n nodes such that the average height of a node is $\Theta(\log n)$ but the height of the tree is $\omega(\log n)$.