CECS 528 Exam 1, Spring 2009, Dr. Ebert

1. Let $c > 1$ be a constant. Establish that $n^c - (n-1)^c = \Theta(n^{c-1})$. (15 points)

2. Let $T_1(n) = \mathrm{O(f(n))}$ and $T_2(n) = \mathrm{O(f(n))}$. Is it true that $T_1(n) + T_2(n) = \mathrm{O(f(n))}$? If yes, prove that it is true by establishing the appropriate C and k constants. If not give a counterexample showing why it is not true. (15 points)

3. The expected running time of a randomized algorithm for generating permutations was determined to be on the order of

$$\frac{n}{n} + \frac{n}{n-1} + \cdots + \frac{n}{2} + \frac{n}{1}.$$

Determine the order of growth of this series. Show work and/or explain. (15 points)

4. Use the substiution method to establish that, if

$$T(n) = T(\lceil \frac{n}{3} \rceil) + T(\lceil \frac{2n}{3} \rceil) + an,$$

where $a > 0$ is a constant, then $T(n) = \Omega(n \log n)$. (20 points)

5. Let $T(n) = 1$ when $n \leq 2$, and $T(n) = T(\sqrt{n}) + 1$, for $n > 2$. Determine the asymptotic growth of $T(n)$ by constructing a recursion tree. What is the height of this tree? (15 points)

6. State the Master Theorem (both the hypothesis and conclusion) and use it to solve the recurrence relation $T(n) = 9T(n/3) + n^2$. (20 points)

7 Recall the Median-of-Five Find Statistic algorithm. Suppose that instead of dividing the input array into groups of size five, we instead used groups of size three. With this change, derive a new upper bound on the running time of $T(n)$. Carefully explain how you derived the new recurrence relation. Based on what you've learned in this course, explain whether or not $T(n) = \mathrm{O(n)}$ still holds true. Hint: using groups of size five, the recurrence relation is $T(n) = T(\lceil \frac{n}{5} \rceil) + T(7n/10 + 6) + an$ (20 points)

8. Let X, Y, and Z be three independent and uniform binary random variables. Let $S = X + Y + Z$. Determine the range space and probability distribution for the random variable $E[S|X]]$. Calculate the expectancy of this random variable in two different ways. (15 points)

9. Show how to multiply the complex numbers $a + bi$ and $c + di$ using only three real multiplications. The algorithm should take a, b, c, and d as input, and produce the real component $ac - bd$ and the imaginary component $ad + bc$. (15 points)